International Journal of Research in Advent Technology, Vol.6, No.11, November 2018
E-ISSN: 2321-9637
Available online at www.ijrat.org

Super Edge Bimagic Lableings of Merging of A Path and Star with Various Cycles of Finite Length

G. A. Mogan[1], M. Kamaraj[2], and A. Solairaju[3]
[3]*Assitant Professor of Mathematics,*
[2]*Associate Professor and Head*
[3]*Associate Professor of Mathematics*
[1]*Dr. Paul's Engineering College, Paul Nagar, Pulichappatham-605 109.*
[2]*Department of Mathematics, Govt. Arts & Science College, Sivakasi-626 124, India.*
[3]*Jamal Mohamed College (Autonomous), Trichy, India, solaijmc@gamil.com*

Abstract: Semanicova [2006] investigated magic and super magic circulant graphs. Gao and Zhang [2008] noted super edge-graceful labelings of caterpilars. Lopez et. al. [2011] initiated bimagic and other generalizations of super edge-magic labeling. Ahmad [2011] highlighted super edge magic deficiency of some families related to ladder graphs. super edge bimagic labelings of merging any star and a path of 4 vertices with cycle having either 5, 6, 7, 8, 9, or 10 vertices.
Keywords: magic labeling, bimagic labeling, edge magic labeling, super edge bimagic labeling.

1. INTRODUCTION

Baskar Babujee [2004] identified bimagic labelings in path graphs. Baskar Babujee and Jagadesh [2008] got super edge bimagic labeling for disconnected graphs like star and wheel. Baskar Babujee and Jagadesh [2008] visualized vertex consecutive edge bimagic labeling for star. Baca et. al. [2007] noted super edge-antimagic of path-like trees. Fukuchi [2001] analyzed edge-magic labelings of generalized Petersen graphs P(n,2). Ivanco and Semanicova [2007] constructed supermagic graphs using antimagic graphs. Liang et. al. [2014] found anti-magic labeling of trees. Ngurah et. al. [2007] discussed (super) edge-magic total labeling of subdivision of $K_{1,3}$. Shiu and Lee [2002] highlighted some edge-magic cubic graphs. Swaminathan and Jeyanthi [2008] approached super edge-magic labeling of some new classes of graphs.

Section 2 – Basic definitions:

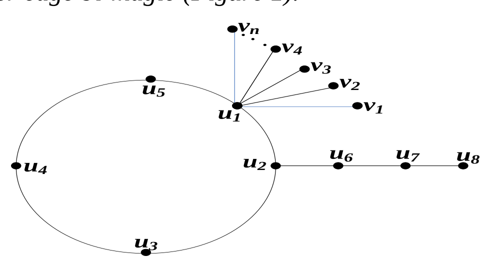
Definition 2.1: A graph G(V,E) with order p and size q is edge magic if there exists a bijection f : V ∪ E → {1, 2,..., p + q} such that f(u) + f(v) + f(uv) is a constant , for all edges uv ∈ E and it is super edge magic if g further satisfies g(V)={1,2,..., p}.

Definition 2.2: A graph G(V,E) with order p and size q is edge bimagic if there exists a bijection g : V ∪ E → {1,2,..., p + q} such that g(u) + g(v) + g(uv) is a constant either c_1 or c_2 for all edges uv ∈ E, and G is super edge bimagic (SEBM) if g also satisfies g(V) = { 1, 2 ,..., p }.

Section 3: Super edge bimagic labelings for $S_n*C_5*P_4$, $S_n*C_6*P_4$ and $S_n * C_7 * P_4$:

Definition 3.1: $S_n * C_5 * P_4$ is a connected graph whose vertex set is {v_1, v_2,.., v_n , $u_{1,..}$.,u_7, u_8} and edge set is {u_1v_i : i = 1 to n } ∪ {u_2u_6, u_6u_7, u_7u_8} ∪ {u_1u_2, u_2u_3, u_3u_4, u_4u_5, u_5u_1}. Here a cycle C_5 of length 5 has vertex set is {u_1,…,u_5}, and edge set is {u_1u_2, u_2u_3, u_3u_4, u_4u_5, u_5u_1} A path P_4 has 3 vertices u_2, u_6, u_7 and u_8, and edge set is {u_2u_6, u_6u_7, u_7u_8}. Finally, S_n is a star graph whose vertex set is {u_1,v_1,v_2,…,v_n} with root vertex u_1, and edge set is{u_1v_i:i=1 to n}.

Theorem 3.2: *The connected graph $S_n * C_5 * P_4$ is super edge bi-magic (Figure 1).*

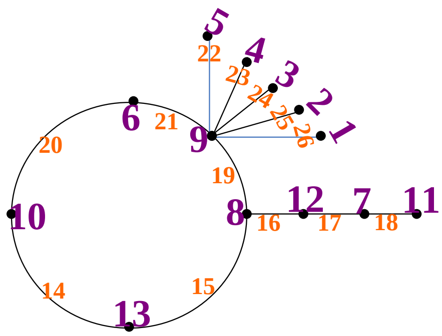


Define f : V(G) → {1, 2,….,n} by
$f(u_5) = n + 1$; $f(u_7) = n + 2$; $f(u_2) = n + 3$; $f(u_1) = n + 4$;
$f(u_4) = n + 5$; $f(u_8) = n + 6$; $f(u_6) = n + 7$; $f(u_3) = n + 8$;
f(v) = i , i = 1 to n.

Define f : E(G) →{p + 1, p + 2,…., p + q}
$f(u_3u_4) = p + 1 = n + 9$; $f(u_2u_3) = n + 10$; $f(u_2u_6) = n + 11$; $f(u_6u_7) = n + 12$;
$f(u_7u_8) = n + 13$; $f(u_1u_5) = n + 14$;
$f(u_4u_5) = n + 15$; $f(u_1u_5) = n + 16$;
$f(u_1vi) = 2n+17- i$, i = 1 to n.

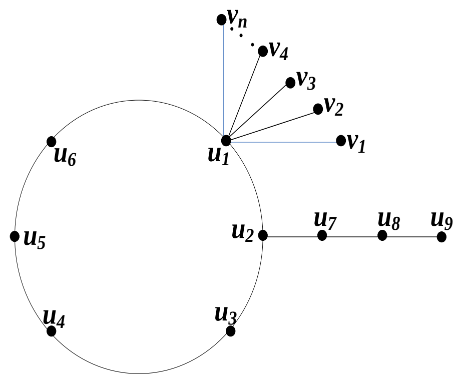
So f satisfies the conditions for super edge bimagic labeling for vertices and edges of the given graph and so $S_n * C_5 * P_4$ is super edge bimagic.

Example 3.3: *Super edge bimagic labelings for $S_5 * C_5 * P_4$ is given (Figure 2).*



Definition 3.4: $S_n * C_6 * P_4$ is a connected graph whose vertex set is $\{v_1, v_2,.., v_n , u_{1,}…,u_7, u_8, u_9\}$ and edge set is $\{u_1v_i : i = 1 \text{ to } n \} \cup \{u_2u_7, u_7u_8, u_8u_9\} \cup \{u_1u_2, u_2u_3, u_3u_4, u_4u_5, u_5u_6, u_6u_1\}$. Here a cycle C_6 of length 6 has vertex set is $\{u_{1,}…,u_{6}\}$, and edge set is $\{u_1u_2, u_2u_3, u_3u_4, u_4u_5, u_5u_6, u_6u_1\}$. A path P_4 has 4 vertices u_2, u_7, u_8 and u_9, and edge set is $\{u_2u_7, u_7u_8, u_8u_9\}$. Finally, S_n is a star graph whose vertex set is $\{u_1,v_1, v_2,…,v_n\}$ with root vertex u_1, and edge set is $\{u_1v_i : i = 1 \text{ to } n\}$.

Theorem 3.5: *The connected graph $S_n * C_6 * P_4$ is super edge bi-magic (Figure 3)*

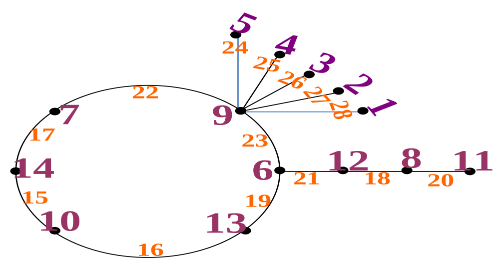


Define f : V(G) → {1, 2,….,n} by
$f(u_2) = n + 1$; $f(u_6) = n + 2$; ; $f(u_8) = n + 3$; $f(u_1) = n + 4$; $f(u_4) = n + 5$; $f(u_9) = n + 6$; $f(u_7) = n + 7$; $f(u_3) = n + 8$; $f(u_5) = n + 9$.
f(v) = i , i = 1 to n.

Define f : E(G) →{p+1, p+2,….,p+q}
$f(u_4u_5) = p + 1 = n + 10$; $f(u_3u_4) = n + 11$; $f(u_5u_6) = n + 12$; $f(u_7u_8) = n + 13$;
$f(u_2u_3) = n+14$;$f(u_8u_9)=n+15$; $f(u_2u_7) = n+16$;
$f(u_6u_1) = n + 17$; $f(u_1u_2) = n + 18$.
$f(u_1vi) = 2n+19- i$, i = 1 to n.

So f satisfies the conditions for super edge bimagic labeling for vertices and edges of the given graph and so $S_n * C_6 * P_4$ is super edge bi-magic.

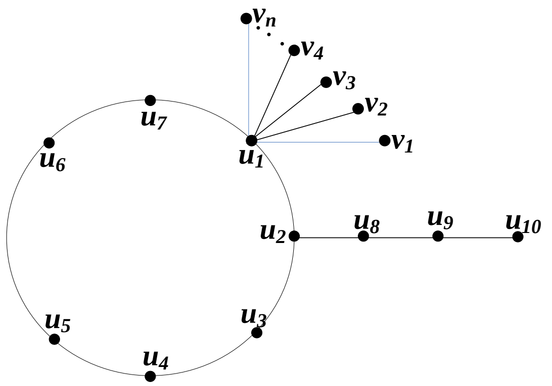
Example 3.6: *Super edge bimagic labelings for $S_5 * C_6 * P_4$ is given (Figure 4).*



Definition 3.7: $S_n * C_7 * P_4$ is a connected graph whose vertex set is $\{v_1, v_2,.., v_n , u_{1,}…,u_7, u_8, u_9, u_{10}\}$ and edge set is $\{u_1v_i : i = 1 \text{ to } n \} \cup \{u_2u_8, u_8u_9, u_9u_{10}\} \cup \{u_1u_2, u_2u_3, u_3u_4, u_4u_5, u_5u_6, u_6u_7, u_7u_1\}$. Here a cycle C_7 of length 7 has vertex set is $\{u_{1,}…,u_7\}$, and edge set is $\{u_1u_2, u_2u_3, u_3u_4, u_4u_5, u_5u_6, u_6u_7, u_7u_1\}$. A path P_4 has 4 vertices u_2, u_8, u_9 and u_{10}, and edge set is $\{u_2u_8, u_8u_9, u_9u_{10}\}$. S_n is a star

graph whose vertex set is $\{u_1, v_1, v_2, \ldots, v_n\}$ with root vertex u_1, and edge set is $\{u_1v_i : i=1 \text{ to } n\}$.

Theorem 3.8: *The connected graph $S_n * C_7 * P_4$ is super edge bi-magic (Figure 5).*

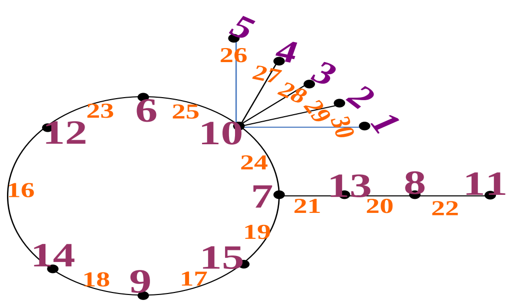


Define $f : V(G) \to \{1, 2, \ldots, n\}$ by
$f(u_7) = n + 1$; $f(u_2) = n + 2$; $f(u_9) = n + 3$; $f(u_4) = n + 4$; $f(u_1) = n + 5$;
$f(u_{10}) = n + 6$; $f(u_6) = n + 7$; $f(u_8) = n + 8$; $f(u_5) = n + 9$; $f(u_3) = n + 10$;
$f(v) = i$, $i = 1$ to n.

Define $f : E(G) \to \{p+1, p+2, \ldots, p+q\}$
$f(u_5u_6) = p + 1 = n + 11$; $f(u_3u_4) = n + 12$; $f(u_5u_5) = n + 13$; $f(u_2u_3) = n + 14$;
$f(u_8u_9) = n + 15$; $f(u_2u_8) = n + 16$; $f(u_9u_{10}) = n + 17$; $f(u_6u_7) = n + 18$; $f(u_1u_2) = n + 19$;
$f(u_7u_1) = n + 20$;
$f(u_1vi) = 2n+21- i$, $i = 1$ to n.

So f satisfies the conditions for super edge bimagic labeling for vertices and edges of the given graph and so $S_n * C_7 * P_4$ is super edge bimagic.

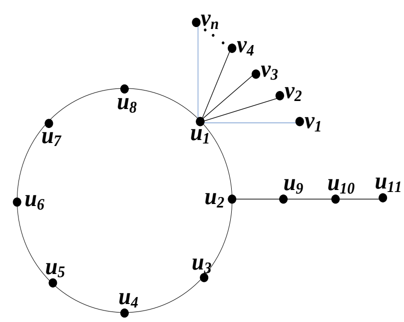
Example 3.9: *Super edge bimagic labelings for $S_5 * C_7 * P_4$ is given (Figure 6).*



Section 4: Super edge bimagic labelings for $S_n * C_8 * P_4$, $S_n * C_9 * P_4$ and $S_n * C_{10} * P_4$:

Definition 4.1: $S_n * C_8 * P_4$ is a connected graph whose vertex set is $\{v_1, v_2, \ldots, v_n, u_1, \ldots, u_7, u_8, u_9, u_{10}, u_{11}\}$ and edge set is $\{u_1v_i : i = 1 \text{ to } n\} \cup \{u_2u_9, u_8u_{10}, u_9u_{11}\} \cup \{u_1u_2, u_2u_3, u_3u_4, u_4u_5, u_5u_6, u_6u_7, u_7u_8, u_8u_1\}$. Here a cycle C_8 of length 8 has vertex set is $\{u_1, \ldots, u_8\}$, and edge set is $\{u_1u_2, u_2u_3, u_3u_4, u_4u_5, u_5u_6, u_6u_7, u_7u_8, u_8u_1\}$. A path P_4 has 4 vertices u_2, u_9, u_{10} and u_{11}, and edge set is $\{u_2u_9, u_9u_{10}, u_{10}u_{11}\}$. Finally, S_n is a star graph whose vertex set is $\{u_1, v_1, v_2, \ldots, v_n\}$ with root vertex u_1, and edge set is $\{u_1v_i : i = 1 \text{ to } n\}$.

Theorem 4.2: *The connected graph $S_n * C_8 * P_4$ is super edge bi-magic (Figure 7)*



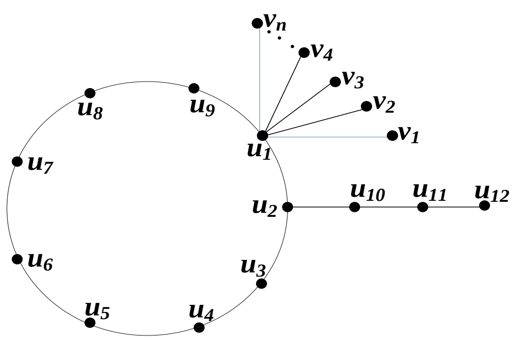
Define $f : V(G) \to \{1, 2, \ldots, n\}$ by
$f(u_8) = n + 1$; $f(u_2) = n + 2$; $f(u_{10}) = n + 3$; $f(u_4) = n + 4$; $f(u_1) = n + 5$;
$f(u_6) = n + 6$; $f(u_{11}) = n + 7$; $f(u_7) = n + 8$; $f(u_9) = n + 9$; $f(u_3) = n + 10$; $f(u_5) = n + 11$;
$f(v) = i$, $i = 1$ to n.

Define $f : E(G) \to \{p+1, p+2, \ldots, p+q\}$
$f(u_5u_6) = p + 1 = n + 12$; $f(u_4u_5) = n + 13$; $f(u_3u_4) = n + 14$; $f(u_6u_7) = n + 15$;
$f(u_9u_{10}) = n + 16$; $f(u_2u_3) = n + 17$; $f(u_2u_9) = n + 18$; $f(u_{10}u_{11}) = n + 19$; $f(u_7u_8) = n + 20$; $f(u_1u_2) = n + 21$;
$f(u_1u_8) = n + 22$;
$f(u_1vi) = 2n + 23- i$, $i = 1$ to n.

So f satisfies the conditions for super edge bimagic labeling for vertices and edges of the given graph and so $S_n * C_8 * P_4$ is super edge bimagic.

Definition 4.3: $S_n * C_9 * P_4$ is a connected graph whose vertex set is $\{v_1, v_2,.., v_n, u_1,…,u_7, u_8, u_9, u_{10}, u_{11}, u_{12}\}$ and edge set is $\{u_1v_i : i = 1 \text{ to } n\} \cup \{u_2u_{10}, u_{10}u_{11}, u_{11}u_{12}\} \cup \{u_1u_2, u_2u_3, u_3u_4, u_4u_5, u_5u_6, u_6u_7, u_7u_8, u_8u_9, u_9u_1\}$. Here a cycle C_9 of length 9 has vertex set is $\{u_1,…,u_9\}$, and edge set is $\{u_1u_2, u_2u_3, u_3u_4, u_4u_5, u_5u_6, u_6u_7, u_7u_8, u_8u_9, u_9u_1\}$. A path P_4 has 4 vertices u_2, u_{10}, u_{11} and u_{12}, and edge set is $\{u_2u_{10}, u_{10}u_{11}, u_1u_{12}\}$. Finally, S_n is a star graph whose vertex set is $\{u_1,v_1, v_2,…,v_n\}$ with root vertex u_1, and edge set is $\{u_1v_i : i = 1 \text{ to } n\}$.

Theorem 4.4: *The connected graph $C_9*P_4*S_n$ is super edge bi-magic (Figure 8)*



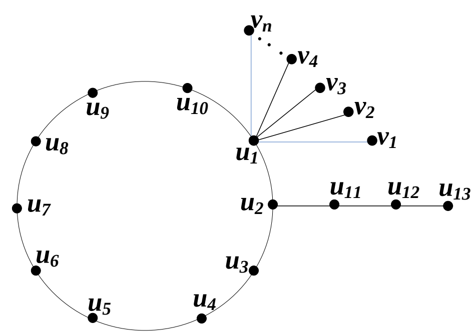
Define $f : V(G) \to \{1, 2,….,n\}$ by
$f(u_9) = n + 1$; $f(u_2) = n + 2$; $f(u_{11}) = n + 3$; $f(u_7) = n + 4$; $f(u_5) = n + 5$;
$f(u_1) = n + 6$; $f(u_3) = n + 7$; $f(u_{10}) = n + 8$; $f(u_8) = n + 9$; $f(u_6) = n + 10$; $f(u_4) = n + 11$; $f(u_{12}) = n + 11$; $f(v) = i$, $i = 1$ to n.

Define $f : E(G) \to \{p+1, p+2,….,p+q\}$
$f(u_3u_4) = p + 1 = n + 13$; $f(u_4u_5) = n + 14$; $f(u_5u_6) = n + 15$; $f(u_{11}u_{12}) = n + 16$;
$f(u_6u_7) = n + 17$; $f(u_7u_8) = n + 18$; $f(u_{10}u_{11}) = n + 19$; $f(u_2u_{11}) = n + 20$; $f(u_8u_9) = n + 21$; $f(u_2u_3) = n + 22$;
$f(u_1u_2) = n + 23$;
$f(u_1u_9) = n + 24$;
$f(u_1vi) = 2n+25- i$, $i = 1$ to n.

So f satisfies the conditions for super edge bimagic labeling for vertices and edges of the given graph and so $S_n * C_9 * P_4$ is super edge bimagic.

Definition 4.5: $S_n * C_{10} * P_4$ is a connected graph whose vertex set is $\{v_1, v_2,.., v_n, u_1,…,u_7, u_8, u_9, u_{10}, u_{11}, u_{12}, u_{13}\}$ and edge set is $\{u_1v_i : i = 1 \text{ to } n\} \cup \{u_2u_{11}, u_{10}u_{12}, u_{11}u_{13}\} \cup \{u_1u_2, u_2u_3, u_3u_4, u_4u_5, u_5u_6, u_6u_7, u_7u_8, u_8u_9, u_9u_{10}, u_{10}u_1\}$. Here a cycle C_{10} of length 10 has vertex set is $\{u_1,…,u_{10}\}$, and edge set is $\{u_1u_2, u_2u_3, u_3u_4, u_4u_5, u_5u_6, u_6u_7, u_7u_8, u_8u_9, u_9u_{10}, u_{10}u_1\}$. A path P_4 has 4 vertices u_2, u_{11}, u_{12} and u_{13}, and edge set is $\{u_2u_{11}, u_{10}u_{12}, u_1u_{13}\}$. Finally, S_n is a star graph whose vertex set is $\{u_1,v_1, v_2,…,v_n\}$ with root vertex u_1, and edge set is $\{u_1v_i : i = 1 \text{ to } n\}$.

Theorem 4.6: *The connected graph $S_n * C_{10} * P_4$ is super edge bi-magic (Figure 9)*



Define $f : V(G) \to \{1, 2,….,n\}$ by
$f(u_{10}) = n + 1$; $f(u_2) = n + 2$; $f(u_{12}) = n + 3$; $f(u_8) = n + 4$; $f(u_6) = n + 5$;
$f(u_1) = n + 6$; $f(u_4) = n + 7$; $f(u_{11}) = n + 8$; $f(u_{13}) = n + 9$; $f(u_9) = n + 10$;
$f(u_7) = n + 11$; $f(u_5) = n + 12$; $f(u_3) = n + 13$.
$f(v) = i$, $i = 1$ to n.

Define $f : E(G) \to \{p+1, p+2,….,p+q\}$
$f(u_3u_4) = p + 1 = n + 14$; $f(u_4u_5) = n + 15$; $f(u_5u_6) = n + 16$; $f(u_6u_7) = n + 17$;
$f(u_7u_8) = n + 18$; $f(u_2u_3) = n + 19$;
$f(u_8u_9) = n + 20$; $f(u_{12}u_{13}) = n + 21$;
$f(u_{811}u_{12}) = n + 22$; $f(u_9u_{10}) = n + 23$;
$f(u_2u_{11}) = n + 24$; $f(u_1u_2) = n + 25$;
$f(u_{10}u_1) = n + 26$;
$f(u_1vi) = 2n+27- i$, $i = 1$ to n.

So f satisfies the conditions for super edge bimagic labeling for vertices and edges of the given graph and so $S_n * C_{10} * P_4$ is super edge bimagic.

REFERENCES

[1] A. Ahmad, I. Javaid, M. F. Nadeem, and R. Hasni, On super edge magic deficiency of some

families related to ladder graphs, Australas. J. Combin., 51(2011) 201-208.
[2] J. Baskar Babujee, Bimagic labelings in path graphs, Math. Education, 38, (2004), 12-16.
[3] J. Baskar Babujee and R. Jagadesh, Super edge bimagic labeling for disconnected graphs, Inter. Journal Appl. Math. Eng. Sci., 2 , (2008), 171-175.
[4] J. Baskar Babujee and R. Jagadesh, Vertex consecutive edge bimagic labeling for star like graphs Global J. Appl. Math. and Math. Sci., 1, (2008), 197-202.
[5] M. Baca, Y. Lin, and F. A. Muntaner-Batle, Super edge-antimagic of path-like trees, Util. Math., 73, (2007), 117-128.
[6] Y. Fukuchi, Edge-magic labelings of generalized Petersen graphs P(n,2), Ars Combin., 59 (2001) 253-257.
[7] Z. Gao and X. Zhang, A note on super edge-graceful labelings of caterpilars, J. Shanghai Jiaotong Univ., 42 (2008) 493-495.
[8] J. Ivanco and A. Semanicova, Some constructions of supermagic graphs using antimagic graphs, SUT J. Math., 42 (2007) 177-186.
[9] Y.-C. Liang, T.-L. Wong, and X. Zhu, Anti-magic labeling of trees, Discrete Math., 331 (2014) 9-14.
[10] S. C. Lopez, F. A. Muntaner-Batle, and M. Rius-Font, Bi-magic and other generalizations of super edge-magic labeling, Bull. Aust. Math. Soc., 84 (2011) 137-152.
[11] A. A. G. Ngurah, R. Simamjuntak, and E. Baskoro, On (super) edge-magic total labeling of subdivision of K1,3, SUT J. Math., 43 (2007) 127-136.
[12] A. Semanicov´a, On magic and supermagic circulant graphs, Discrete Math., 306 (2006) 2263-2269.
[13] W. C. Shiu and S. M. Lee, Some edge-magic cubic graphs, J. Combin. Math. Combin. Comput., 40 (2002) 115-127.
[14] V. Swaminathan and P. Jeyanthi, Super edge-magic labeling of some new classes of graphs, Math. Education, XLII (2) (2008) 91-94.